Game Theory

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1. Introduction:

The theory of Games was developed by John Von Neumann and Oskar Morgenstern in their famous work entitled "The Theory of Games and Economic Behavior" in 1944. This theory determines the standards of rational behaviour where the outcomes depend on the action of interdependent individuals (firms).

A competitive situation is called a *game*. We say that a competitive situation exists if two or more individuals make decisions in a situation that involves conflicting interests and in which the outcome is controlled by the decision of all the concerned parties. It is obvious that term 'game' represents a conflict between two or more parties. A situation is termed a game when it possesses the following properties:

1. The number of competitors (players) is finite.
2. There is a conflict in interests between the competitors.
3. Each of the competitors has generally a finite set of possible courses of action.
4. The rules governing these choices are specified and known to all players, a play of the game results when each of the players chooses a single course of action from the list of courses available to him.
5. The outcome of the game is affected by the choices made by all the players.
6. The outcome for all specific set of choices by all the players is known in advance and numerically defined.

Game theory is a type of decision theory which is based on reasoning in which the choice of action is determined after considering the possible alternatives available to the opponents playing the same game. Game theory has been projected as a scientific approach to rational decision making, and rightly so.

2. Applications of Game Theory:

Economics is the biggest customer for the ideas churned out by game theorists and the fact is that game theory has found ready applications in economics. For example, the concept of *monopoly* is simple from a game-theoretic perspective in the sense that it can be treated as a game with only one player.

Game theory has applications also in the domain of Political Science, Biology and Social Philosophy to a varied degree. (If you are interested more in knowing about the applications of game theory, read *Ken Binmore's* classic, Fun *and Games – A Text on Game Theory*)

3. Strategy:

It is defined as a complete set of plans of action specifying precisely what the player will do under every possible future contingency that might occur during the play of the game, i.e., strategy of a player is a decision rule he uses for making a choice from his list of courses of action.

The players in the game strive for *optimal* strategies. An optimal strategy is such that it provides the best situation in the game in the sense that it involves maximal pay-offs to the players.

Strategy may be classified as:
Pure Strategy: A strategy is called pure if one knows in advance that it is certain to be adopted irrespective of the strategy the other players might choose.

Mixed Strategy: A mixed strategy represents a combination of two or more strategies that are selected one at a time, according to pre-determined probabilities. Thus, in employing a mixed strategy, a player decides to mix his choices among several alternatives in a certain ratio.

4. Payoff:

It is the outcome of playing the game, or in other words, the result of adopting a particular strategy.

A payoff matrix is a table showing the amount received by the player named at the left hand side after all possible plays of the game. The payment is made by the player named at the top of the table. As is evident, a payoff matrix may be constructed only for *two-person* games. If player A has *m* strategies and player B has *n* strategies, the payoff matrix will be of the order m x n.

5. Game Models:

There are several game theory models which can be classified on the basis of factors like the number of players involved, the sum of gains or losses, and the number of strategies employed in the game.

If there are only two participants, we have a *two-person game*. As a general case, for *n* participants, we have an *n-person game*. It is to be remembered that *person* does not imply individuals only but may mean *categories*, and members of each category share identical interests. For example, a game of football is not a 22–player game but a 2-player game.

If in a game, the sum of gains and losses for all the participants considered together is equal to zero, it is called a *zero-sum* game or *constant-sum* game. Otherwise, we will have a *non-zero-sum* game.

Based on the number of strategies available, a game may also be termed as *finite* or *infinite*. But modeling infinite games is a difficult proposition.

We may also have *combination* classification, for e.g., a *two-person zero-sum* game. For a two-person game again, we may have 2 x n and m x 2 games, which means that in the first case, player A has 2 strategies and player B has n strategies. So, theoretically, as a general rule, we should have m x n games also (Eureka! What a discovery!).

6. Maximin – Minimax Principle and Saddle Points:

In a two-person zero-sum game, say, we have two players A and B. If we look from A's perspective, we may safely assume that A wishes to maximize his gain while B would like to minimize his losses. So, whenever A adopts a strategy to maximize his gains, B would adopt a counter strategy with the objective of minimizing A's gain or in other words, minimizing his own losses.

The problem is to determine the best strategy for A and B, assuming that both are acquainted with the information contained in the pay-off matrix and that each one is not aware of the move the other is likely to take.

From the discussion above, it is clear that we are in a situation where A would go for maximizing his minimum gains while B would strive for minimizing his maximum losses. So, A is obtaining a value called the *maximin* value and the corresponding strategy is called the Maximin strategy. Similarly, B is obtaining a value called the *minimax* value and the corresponding strategy is called the Minimax strategy. In a payoff matrix, the maximin and minimax values are found out by finding the maximum value of the row minimas and minimum value of the column maximas respectively. When maximin value = minimax value, the corresponding strategies are called optimal strategy and the game is said to have a *saddle point.* A saddle point may therefore be defined as a position in the payoff matrix where the maximum of row minimas coincides with the minimum of the column maximas. The payoff at the saddle point is called the *value* of the game. Existence of one saddle point will mean the existence of pure strategies, as is clear from the definition of pure strategies given above,

Based on the concept of maximin and minimax, we have another classification of games. A game is said to be *fair* if maximin value = minimax value = 0 (Otherwise, term it as unfair. Anyway in the real world, things are seldom fair!). Actually, a game is said to be *strictly determinable* if maximin value = minimax value ≠ 0.

Continuing on saddle points, there may be more than one saddle point or there might be none. Having more than one saddle point should also mean the absence of pure strategies (This is what I think, what do you say?). My logic simply says that if there is more than one saddle point, a player has the choice of adopting more than one strategy, countering the definition of a pure strategy (Unfortunately again, I am prone to making mistakes, so I am not making a statement per se!). Of course, it is possible to have more than one pure strategy but at different levels, or I should say for different moves at different points of time of the game.

If there are no saddle points, we have a situation of mixed strategies being adopted by the players in a certain pre-ordered ratio.

7. Dominance Rule:

In a game, sometimes a strategy available to a player might be found to be preferable to some other strategy / strategies. Such a strategy is said to *dominate* the other one(s). This concept is useful in simplifying games and thus it helps in solving games with lesser effort.

The rule calls for eliminating rows and columns from the given payoff matrix. We may eliminate a particular row, if every element in that row is less than the corresponding element of another row. Eliminate a column if every element in that column is greater than the corresponding elements of another column.

A's Strategy	B's Strategy			
		B1	B2	B3
	A1	12	-8	-2
	A2	6	7	3
	A3	-10	-6	2

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In the above payoff matrix, we eliminate row 3 (A3) and thereafter eliminate column1 (B1). All elements of row3 are less than the corresponding elements of row2. After eliminating the row, we see that all elements of column1 are greater than the corresponding elements of column3. The logic followed is simple – A shall never prefer to play A3, because, in comparison to this strategy, it shall be better off by adopting A2 regardless of what strategy is adopted by B. A similar argument holds in case of eliminating B1.

If saddle point of the game exists, the game shall naturally be reduced to a 1 x 1 game. So, the property of dominance can be used in place of the maximin – minimax principle to verify the existence of a saddle point. Simply great!

8. Limitations of Game Theory:

The discussion was restricted to the two-person zero sum games. There are practically no applications of game theory to the real world situations as of now. But advances are being made towards that direction. This is because of the assumption underlying the theory. A partial list of limitations is given below:

- Finite number of strategies
- Complete knowledge about the strategies of the participants
- Zero outcomes
- Finite number of competitors
- No consideration about the risk and uncertainty involved
- Certainity of payoff

9. Conclusion:

A great deal of nonsense has been written about game theory and may be this small write-up is no exception. It is true, as Bertrand Russell said of philosophy, that reading game theory is like reading a fairy tale – you must absolve yourself of reality while the tale is being told if you are to appreciate what is going on. But don’t forget to reinvent your skeptical attitude once the story is over! Be unreasonable, dear! And, do forgive me for trying to get into your head something not as nice as game theory. What to do – I am paid for that!