

Assignment 2
Course Code (Name) : BM206(Quantitative Techniques)

Section 1: Problems

Course Covered : Differentiation, Maxima & Minima, Integration

Instructions : Attempt all questions on your own at the first instance. If you cannot do them on your own, you are free to take help of your batch mates or your course instructor. The assignments need not be submitted but the course instructor reserves the right to ask for the solutions any time.

1. If $y = \log \log \log x^3$, find dy/dx .

2. Find dy/dx when

(i) $\sin(xy) + \frac{x}{y} = x^2 - y$

(ii) $x^y = y^x$

(iii) $x = \log t + \cos t, y = e^t + \sin t$

3. Differentiate $\log x$ w.r.t $\cos x$.

4. If $y = e^{ax}\cos bx$, show that $\frac{d^2y}{dx^2} - 2a \frac{dy}{dx} + (a^2 + b^2)y = 0$

5. Find the maximum and minimum values of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ on $[0,3]$.

6. Show that $(x + 1/x)$ has a maximum and minimum, but the maximum value is less than the minimum value. (A Paradox, you see!)

7. Find the maximum profit that a company can make, if the profit function is given by $p(x) = 41 + 24x - 18x^2$.

8. The revenue SystemX Inc. earns from its *data analysis software* DATALYSER® is related to the advertising (a) and the units sold (q) as per the following function:

$$R = 520 - 5a^2 + 21a + 15qa - 4.5q^2 + 15q$$

The budget constraint is given as $2q + a = 10$. Determine the value of q and a that maximizes revenue subject to the budget constraint.

9. If $f(x, y) = x^3e^{y^2}$, find the first and second order partial derivatives at $(x, y) = (1, 0)$.

10. Determine the value of x and y for which

$$Z = 4x^2 + 2y^2 + 10x - 6 - 4xy$$

is an optimum. Specify whether the optimum is maxima or minima and calculate the value of the function at the optimum.

11. Integrate:

(i) $\int \frac{2x^2 + x - 2}{x - 2} dx$

(ii) $\int \frac{2x + 5}{x^2 + 5x + 9} dx$

(iii)
$$\int_0^4 \frac{1}{x^2} e^{-\frac{1}{x}} dx$$

12. If $f'(x) = 3x^2 - 2/x^3$, and $f(1) = 0$, find $f(x)$.

13. The marginal cost function is given by $MC = 2 + 5e^x$. Find C if $C(0) = 100$.

Section 2: Think???

1. The forest officer posted at a Lion Sanctuary was asked to make a quick assessment of the lion population in the sanctuary. Over a period of two days, he made field visits and tranquilized 20 lions and marked them. After this, over the subsequent two days, he tranquilized 42 lions and found that only 3 lions were marked. Can you think, what was the figure; the forest officer came up with? How far was he correct?

2. You have heard about *price elasticity of demand* in economics. What is its relationship with differentiation? Why is it so named?

Using your understanding, solve the following problem:

Assume that the quantity demanded for a particular commodity is given by the formula

$$D(p) = 8000p^{-1.5}$$

Compute the elasticity of $D(p)$ and find the percentage change in quantity demanded when the price increases by 1% from $p = 4$.