Mathematical & Statistical Tools for Financial Decision Analysis

Himadri Barman

Linear Programming:
Linear Programming deals with the optimization of a function of variables known as objective functions. It is subject to a set of linear equalities and/or inequalities known as *constraints*. Linear Programming is a mathematical technique which involves the allocation of limited resources in an optimal manner, on the basis of a given criterion of optimality.

Linear programming has become nowadays, the mathematical technique most used in solving a variety of problems related with management, from scheduling, media selection, financial planning to capital budgeting, multiple plant location studies, transportation and many others, with the special characteristic that linear programming expect always to maximize or minimize some quantity.

Advantages and Disadvantages of Linear Programming:
First of all it is known that one of the main advantages of linear programming is that it fits strictly with reality or models reality to a considerable extent. In frequent occasions some variables are ignored and hence the problem becomes less rigorous and loses accuracy and certainly, that becomes a disadvantage. Furthermore the model is static which means that it does not consider the changes and the evolution of variables as time goes by. Another obstacle arises in the formulation process where values should be taken into account - they must be known with certainty.

It is also obvious that linearity is an important bound; it means that each decision variable appears in a separate term and has an exponent of 1, so that non-lineal function cannot be used. There may be another two problems consisting of numerous optimal solutions, this is not a simple matter despite the fact could seem a minor concern, and the other problem could be infeasibility. When no solution to the linear programming problem satisfies all the constraints, feasible region does not exist and therefore any solution cannot be reached.

But there still are more advantages; linear programming analysis can help both with determining whether management's plans are feasible and in unbounded cases where the value of the solution is infinitely large, without violating any of the constraints, warning us that the problem is improperly formulated. The ability to analyze as to what happens when we change the values of the objective function or in the constraints is another advantage of using linear programming. We can check easily how the results vary when we change the old coefficients for others. This is called sensitivity analysis, which determines how changes affect the optimal solution to the original linear programming problem. The range of values over which the current solution will remain optimal, despite the change of the coefficients, is called range of optimality. It must be mentioned, another remarkable characteristic of linear programming problems such as, the adapting facility to reality, which allows solving, by computer programs, problems with thousands of variables and constraints.

In conclusion, if we evaluate the pros and cons it can be ascertained that it is not coincidence that linear programming is the most used program in the management area. Despite having several arguments against, there are sound reasons which take us to select this method solving management problems owing to the complexity of the problems that can be handled.

Discriminant Analysis (DA)
Discriminant function analysis is used to determine which variables discriminate between two or more naturally occurring groups. For example, a researcher may want to investigate which variables discriminate between fruits eaten by (1) primates, (2) birds, or (3) squirrels. For that purpose, the researcher could collect data on numerous fruit characteristics of those species eaten by each of the animal groups. Most fruits will naturally fall into one of the three categories. *Discriminant Analysis* could then be used to determine which variables are the best predictors of whether a fruit will be eaten by birds, primates, or squirrels. Take another example. An educational researcher may want to investigate which variables discriminate between high school graduates who decide (1) to go to college, (2) to attend a trade or professional school, or (3) to seek no further training or education. For that purpose the

researcher could collect data on numerous variables prior to students' graduation. After graduation, most students will naturally fall into one of the three categories. *Discriminant Analysis* could then be used to determine which variable(s) are the best predictors of students' subsequent educational choice. The examples above should give you an idea as to where DA may be used in the area of financial decision analysis. It has been used in various areas such as “Prediction of Corporate Bankruptcy”, “Credit Risk”, “Performance of Financial Instruments”, etc.

DA is an earlier alternative to logistic regression, which is now frequently used in place of DA as it usually involves fewer violations of assumptions (independent variables needn't be normally distributed, linearly related, or have equal within-group variances), is robust, handles categorical as well as continuous variables, and has coefficients which many find easier to interpret. Logistic regression is preferred when data are not normal in distribution or group sizes are very unequal. However, discriminant analysis is preferred when the assumptions of linear regression are met since then DA has more statistical power than logistic regression (less chance of type 2 errors - accepting a false null hypothesis).

Simulation:
The word simulation is derived from assumed sequence of occurrences that is something which is apparent than real. For example, war games conducted at great expenses by armed forces are simulated without real enemies and casualties and destructions are only symbolically represented. Anticipated business processes may be similarly simulated. We define simulation as a quantitative procedure which describes a process by developing a model of that process and then conducting a series of organized trial and error experiments to predict the behaviour of the process over time. Simulation is of mainly two types – (i) Analogue (environmental) and (ii) Computer (system) simulation.

Why use simulation? The simple answer is that is that it transfers work to the computer. Models can be handled which have greater complexity, and fewer assumptions, and a more faithful representation of the real-world than those that can be handled tractable by pure mathematical analysis are possible. By changing parameters we can examine interactions, and sensitivities of the system to various factors. Experimenters may either use a simulation to provide a numerical answer to a question, assign a price to a given asset, identify optimal settings for controllable parameters, examine the effect of exogenous variables or identify which of several schemes is more efficient or more profitable. The variables that have the greatest effect on a system can be isolated. We can also use simulation to verify the results obtained from an analytic solution. For example many of the tractable models used in finance to select portfolios and price derivatives are wrong. They put too little weight on the extreme observations, the large positive and negative movements (crashes), which have the most dramatic effect on the results. Is this lack of fit of major concern when we use a standard model such as the Black-Scholes model to price a derivative? Questions such as this one can be answered in part by examining simulations which accord more closely with the real world, but which are intractable to mathematical analysis. Simulation is also used to answer questions starting with “what if”. For example, what would be the result if interest rates rose 3 percentage points over the next 12 months?

The major reasons for applying simulation to financial problems may be listed as below:
1. The actual environment is difficult to observe in reality
2. It may not be possible to develop an analytical solution of the problem
3. Actual observation of the financial system is prohibitively expensive and time consuming
4. There is no sufficient time available to allow the financial system to operate extensively
5. Actual operation and observation of a financial system may be too disruptive

Financial Simulations requires realistic assumption about three critical inputs:
1. An assumed value for the future average return on each asset (i.e., mean)
2. An estimate of the variability of the returns around the mean (i.e., standard deviation) as a measure of risk
3. An assumption about how the portfolio assets will co-vary, or react to changes in each other over time (i.e., correlation)

Financial planners employ one of the two statistical procedures to generate estimates of future asset values – historical simulation and Monte Carlo simulation. Historical simulation methods generate return scenarios by assuming that past events repeat in some chronological fashion. The premise is that potential changes in asset returns will not range beyond the changes previously observed in those assets' values over a defined historical period. Monte Carlo simulation methods, on the other hand, randomly generate future returns by specifying the likelihood that an asset will take a certain future value. To derive such probabilities, Monte Carlo uses the Mean and Variance that characterize a historical sample to impose explicit distributional assumptions in the simulation process. The two alternatives can lead to different expectations of investment risks.

References:

[1] http://html.rincondelvago.com/linear-programming.html assessed on 28.04.2007
[2] http://www.statsoft.com/textbook/stdiscan.html assessed on 27.4.2007
[3] http://www2.chass.ncsu.edu/garson/pa765/discrim.htm assessed on 27.04.2007
[4] http://userwww.sfsu.edu/~efc/classes/biol710/discrim/discrim.pdf assessed on 07.04.2007
[5] Kalavathy, S, *Operations Research,* 2nd Edition, Vikas Publishing House Private Limited, New Delhi – 110 014
[6] http://www.stats.uwaterloo.ca/~dlmcleis/s906/chapt1-6.pdf assessed on 01.05.2007
[7] http://www.vanguard.com.au/library/pdf/Financial%20Simulation%2009%202003.pdf assessed on 01.05.2007
[8] Sharma, J K, *Quantitative Techniques for Managerial Decisions,* 1st Edition, MacMillan India Limited, New Delhi